DIGITAL SIGNAL PROCESSING. Jan 8th.

1. Introduction to DSP: (Ch. 2, textbook)

1.0 Signals, systems & signal processing

- SIGNAL: a physical quantity that varies with time, space and any other independent variables.

  For example: $x_1(t) = 5t^2$, $x_2(t) = 7t$ } two signals varying with time, t.

  A speech signal may be represented as $x(t) = \sum_{i=1}^{N} A_i(t) \sin\{2\pi f_i t + \phi_i\}$;

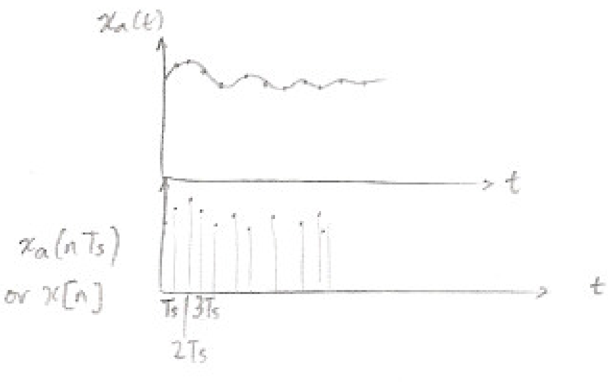
  → sum of several sinusoids with different amplitudes, frequencies and phases.

  Signals carry information which may be useful or unuseful (noise: unwanted)

  Eg. an electrocardiogram (ECG) signal provides a doctor with information about the condition of a heart

  A signal that is a function of two independent variables is an image signal. The independent variables are spatial coordinates

  SIGNALS: → analog (continuous-time) signal
  → discrete-time (or digital) signal → sampled version of analog signal.



Ts: Sampling time.

- SYSTEM: may be a device that forms an operation on a signal. For example, a filter used to reduce the noise from a corrupted signal is a system.

- SIGNAL PROCESSING: an operation designed for extracting, enhancing, storing and transmitting useful information.

TWO WAYS:

(a) analog signal processing (ASP)
↳ performed using electrical networks containing active and passive elements

(b) digital signal processing (DSP)
↳ system using a digital signal processor can be developed by
Software running on a general purpose computer.
e.g. a filter.

ASP has limited scope of performing complicated signal processing applications, whereas DSP is relatively convenient to develop and test. Moreover, the software for the DSP system is portable.

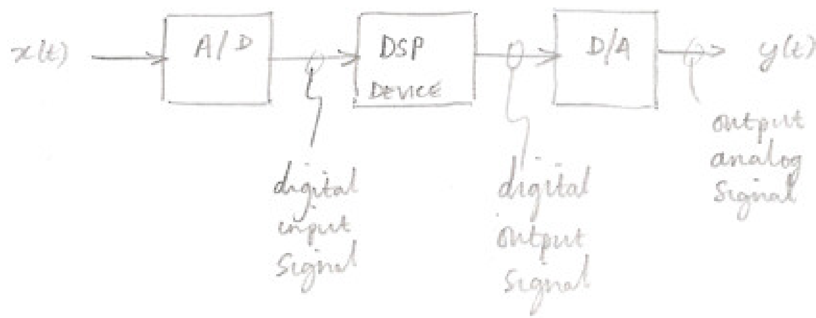


FIG: Block DIAGRAM (dgm) OF A DSP SYSTEM

1.1. Discrete-time signals (Sequence):

↳ defined mathematically as a sequence of numbers.

e.g. n^{th} number (or sample) in a sequence, $x[n] = x_a(nT_s)$
$-\infty < n < \infty$, n is an <u>integer</u>.

($x_a(nT_s)$: value of analog signal at n^{th} sampling instant.)

"The whole seq. $x(n)$" $= \{0.5, 1, -1, 2, 0.5, 0.7, \cdots\}$

(0.5 ↑ $n=0$, 1 ↑ $n=1$, −1 ↑ $n=2$ …)

$x[n] = \{a, b, c, d, e\}$

(a ↑ $n=0$; values to the left are for $n<0$)

The whole sequence may be represented as $\{x[n]\}$

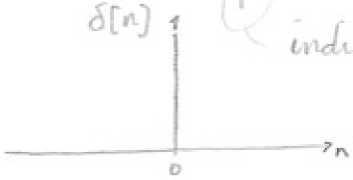
CONVENTION:
For simplicity, we will use $x[n]$ for the whole sequence
For specific samples: $x[1]$, $x[2]$
($x[1]$: 1^{ST} ($n=1$) sample; $x[2]$: 2^{nd} ($n=2$) sample.)

1.1.1: Types of Sequences:

(a) Unit sample or unit impulse or impulse sequence, $\delta[n]$:

Mathematically: $\delta[n] = \{ \underset{\uparrow}{1}, 0, 0, 0, \ldots \}$ ← indicates $n=0$ sample.

Graphically:

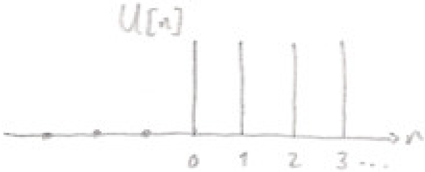


Functionally: $\delta[n] = \begin{cases} 1, & n=0 \\ 0, & n \neq 0 \end{cases}$

(b) Unit step sequence, $U[n]$:

Mathematically: $U[n] = \{ \underset{\uparrow}{1}, 1, 1, 1, \ldots \}$

Graphically:



Functional representation: $U[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$

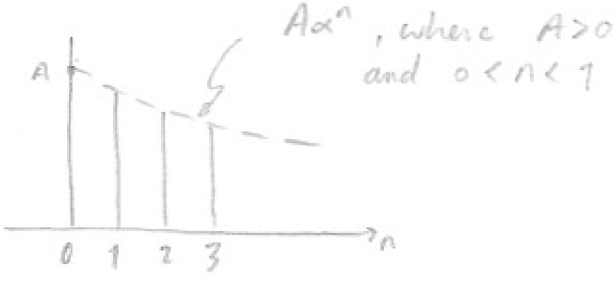
(c) Unit ramp sequence, $U_r[n]$

$U_r[n] = \{ \underset{\uparrow}{0}, 1, 2, 3, 4, \cdots \}$

$U_r[n] = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases}$

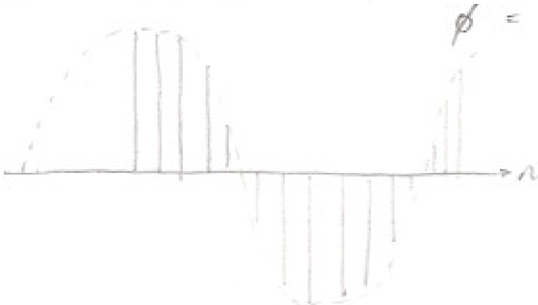
(d) Real exponential seq.

$x[n] = A\alpha^n$



(e) Sinusoidal seq.

$x[n] = A\cos(\omega_0 n + \phi)$, where ω_0 = freq. of complex sinusoid (rad/sample)
ϕ = phase angle



(f) Random sequence → cannot be defined mathematically.

(g) Periodic seq. → a sequence is periodic if $x[n] = x[n+N]$;
(repitition after every N samples).

The smallest value of N for which $x[n] = x[n+N]$ is valid is called the fundamental period of $x[n]$

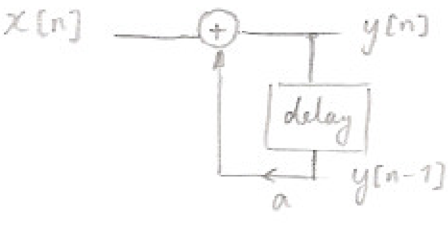
A periodic sequence is denoted by $\tilde{x}[n]$ "tilde"

(h) Complex exponential seq.

$x[n] = A e^{j(\omega_0 n + \phi)}$, similar to sinusoidal sequence

(i) Recursive seq.

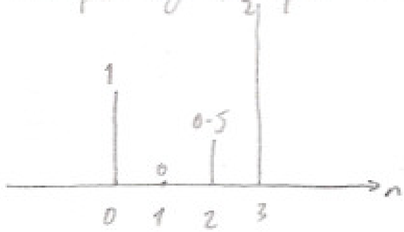
$y[n] = a\,y[n-1] + x[n]$ → present sample depends on the previous samples.



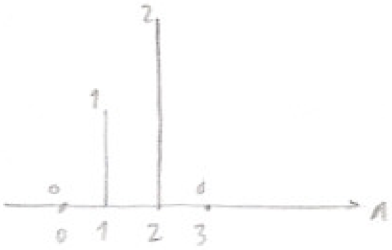
1.1.2: Operations on sequences.

(a) Addition : corresponding sample-by-sample addition

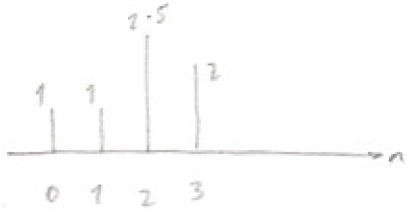
eg. $x_1[n]$



\+ $x_2[n]$

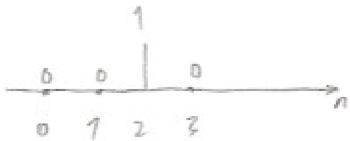


$x_1[n] + x_2[n] \Rightarrow$



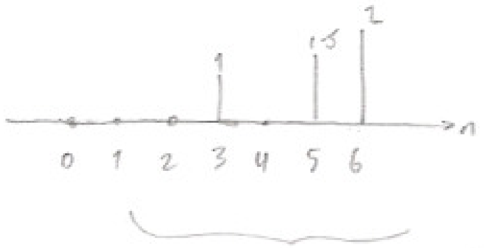
(b) Multiplication : corresponding sample-by-sample multiplication. ($x_1[n]x_2[n]$)

$x_1[n]\,x_2[n] \Rightarrow$



(c) Shifting: $x[n-k]$: $x[n]$ is shifted to the right by k samples

eg. $x_1[n-3] \Rightarrow$



$x_1[n]$ is shifted right by 3 samples.

eg. $x_1[n+3]$ ← shift <u>left</u> by 3 places